



HERON'S FORMULA DERIVATION

Equation 1

$$\text{Area} = \frac{b \cdot h}{2}$$

From triangle ABD...

$$x^2 + h^2 = c^2$$

Equation 2

$$x^2 = c^2 - h^2$$

Equation 3

$$x = \sqrt{c^2 - h^2}$$

From triangle CBD...

$$(b - x)^2 + h^2 = a^2$$

$$(b - x)^2 = a^2 - h^2$$

Equation 4

$$b^2 - 2bx + x^2 = a^2 - h^2$$

Substitute  $x$  and  $x^2$  from Equations 3 and 2 into Equation 4 ...

$$b^2 - 2b\sqrt{c^2 - h^2} + (c^2 - h^2) = a^2 - h^2$$

$$b^2 + c^2 - a^2 = 2b\sqrt{c^2 - h^2}$$

Square both sides ...

$$(b^2 + c^2 - a^2)^2 = 4b^2(c^2 - h^2)$$

$$\frac{(b^2 + c^2 - a^2)^2}{4b^2} = c^2 - h^2$$

$$h^2 = c^2 - \frac{(b^2 + c^2 - a^2)^2}{4b^2}$$

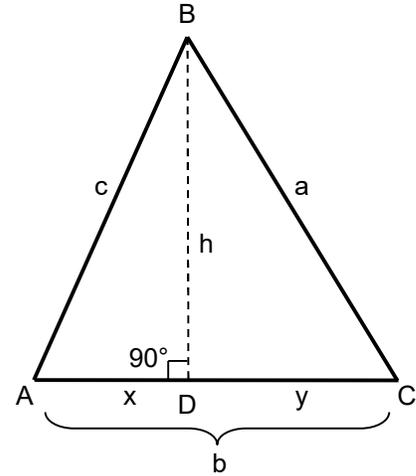
$$h^2 = \frac{4b^2c^2 - (b^2 + c^2 - a^2)^2}{4b^2}$$

$$h^2 = \frac{(2bc)^2 - (b^2 + c^2 - a^2)^2}{4b^2}$$

$$h^2 = \frac{[2bc + (b^2 + c^2 - a^2)] \cdot [2bc - (b^2 + c^2 - a^2)]}{4b^2}$$

$$h^2 = \frac{[2bc + b^2 + c^2 - a^2] \cdot [2bc - b^2 - c^2 + a^2]}{4b^2}$$

$$h^2 = \frac{[(b^2 + 2bc + c^2) - a^2] \cdot [a^2 - (b^2 - 2bc + c^2)]}{4b^2}$$



$$h^2 = \frac{[(b+c)^2 - a^2] \cdot [a^2 - (b-c)^2]}{4b^2}$$

$$h^2 = \frac{[(b+c)+a] \cdot [(b+c)-a] \cdot [a+(b-c)] \cdot [a-(b-c)]}{4b^2}$$

$$h^2 = \frac{(b+c+a)(b+c-a)(a+b-c)(a-b+c)}{4b^2}$$

$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

$$h^2 = \frac{(a+b+c)(a+b+c-2a)(a+b+c-2b)(a+b+c-2c)}{4b^2}$$

Since  $P = a + b + c \dots$

$$h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

Equation 5

$$h = \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

Substitute  $h$  from Equation 5 into Equation 1 ...

$$Area = \frac{1}{2}b \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$Area = \frac{1}{4} \sqrt{P(P-2a)(P-2b)(P-2c)}$$

$$Area = \sqrt{\frac{1}{16} P(P-2a)(P-2b)(P-2c)}$$

$$Area = \sqrt{\left(\frac{P}{2}\right) \left(\frac{P-2a}{2}\right) \left(\frac{P-2b}{2}\right) \left(\frac{P-2c}{2}\right)}$$

$$Area = \sqrt{\frac{P}{2} \left(\frac{P}{2} - a\right) \left(\frac{P}{2} - b\right) \left(\frac{P}{2} - c\right)}$$

Thus...

Given the three sides of a triangle ( $a$ ,  $b$ , and  $c$ ) ...

The area of the triangle is:

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Where the semi perimeter is:

$$s = P/2 = (a + b + c)/2$$